FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
FULL TEST – VII
PAPER –1
TEST DATE: 28-04-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

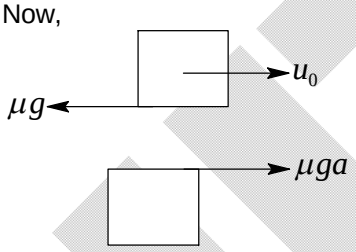
SECTION – A

1. AC

Sol. $\alpha M u_0 + 0 = v'(2\alpha M + M)$

$$v' = \frac{\alpha u_0}{1+\alpha}$$

Now,

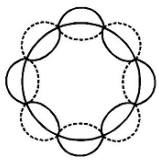


$v' = u_0 - \mu g t$

$v'^2 = 0 + 2\mu g \alpha s$

2. A, B, C, D

Sol. $\frac{8\lambda}{2} = 2\pi R$



$4\lambda = 2\pi R \Rightarrow n\lambda$

$n = 4$

So given excite state = n = 4.

(A) Change in angular momentum = $\frac{(n_2-n_1)h}{2\pi}=\frac{(4-1)\pi}{2\pi}=\frac{3h}{2\pi}$

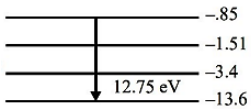
(B) $n\lambda = 2\pi R_n$

$\lambda = 2\pi R_1$

$4\lambda = 2\pi R_4$

$\frac{4\lambda_1}{\lambda_2}=\frac{R_4}{R_1}=\frac{(4)^2}{(1)^2}\Rightarrow\frac{\lambda_4}{\lambda_1}=\frac{4}{1}$

(C)



(D) $\frac{T_4}{T_1}=(4)^3$

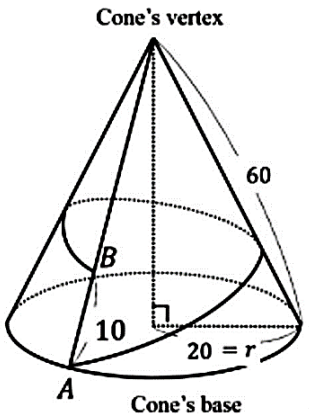
$T \propto n^3$

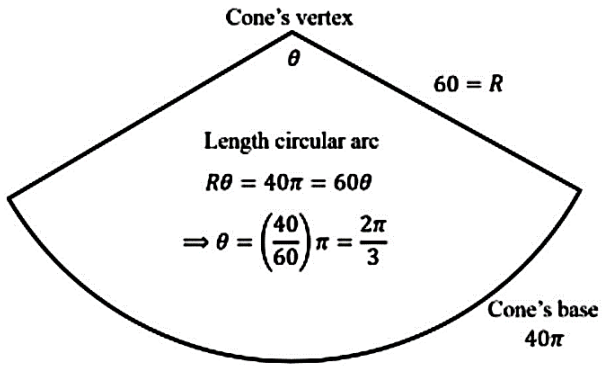
$T=\frac{2\pi r}{v}\Rightarrow n^2\times n$

$T\propto\frac{n^3}{z^2}$

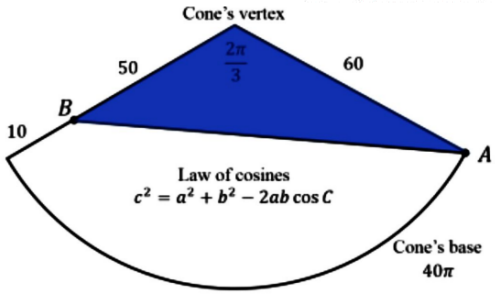
3. A, B

Sol.





Cone's base radius is 20, so circumference = $2\pi r = 2\pi(20) = 40\pi$

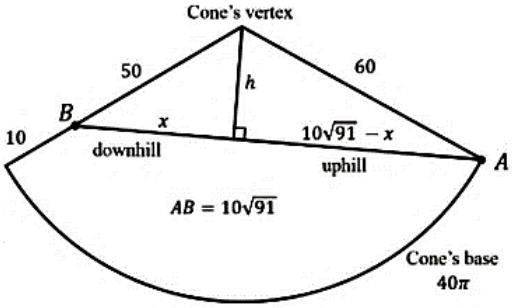


$AB^2=60^2+50^2-2(60)(50)\cos\frac{2\pi}{3}$

$AB^2=9100$

$AB=10\sqrt{91}$

By the Pythagorean Theorem:



$$\left(10\sqrt{91}-x\right)^2+h^2=60^2$$

$$-\qquad x^2+h^2=50^2$$

$$9100-2\left(10\sqrt{91}\right)x=60^2-50^2$$

$$\left(20\sqrt{91}\right)x=8000$$

$$x=\frac{400}{\sqrt{91}}$$

4. A

Sol. From screw gauge z. e. = + 0.20 mm
Dia of chalk = 3.5 – 0.2 = 3.3 mm = 0.33 cm
For vernier callipers, z. e. = - 0.05 cm
Thus, it must show 0.33 – 0.05 = 0.28 cm
So, (A) is correct.

5. B

Sol. Ray will emerge out of the medium up to the critical angle fraction of light escaping from medium is.

Fraction escaping = $\frac{2\pi\left(1-\cos\theta\right)R^2}{4\pi R^2}$

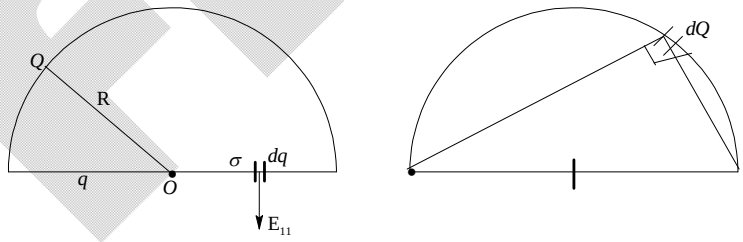
$$\%\frac{1-\cos\theta}{2}$$

Where θ is critical angle which does not depend on depth.

$\therefore a=b$

6. D

Sol.



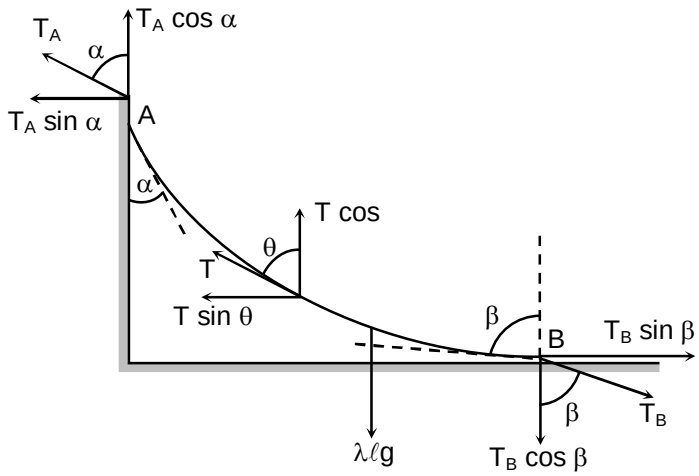
$F=\int dqE_{11}=\sigma\int E_{11}dA=\frac{q}{L2R}\phi$ where ϕ = electric flux due to 'Q' through plane [L×2R]

$$d\phi=\frac{dQ}{4\in_0}\left(\therefore angle\,is\,semicircle=90°=\frac{360°}{4}\right)$$

$$\phi = \frac{Q}{4\in_0} \Rightarrow F = \frac{q}{L2R}\frac{Q}{4\in_0} = \frac{Qq}{8\in .RL}$$

7. C

Sol.



$$T\sin\theta = T_B\sin\beta = \text{constant}$$

$$T\cos\theta = T_B\cos\beta + \lambda\ell\,\text{g}$$

$$T_A\sin\alpha = T_B\sin\beta$$

$$T_A\cos\alpha > T_B\cos\beta$$

$$\tan\alpha < \tan\beta$$

$$\alpha < \beta$$

8. D

Sol. (P) $\dfrac{\Delta Q}{\Delta t} = \dfrac{k_0\left[\pi(2R)^2 - \pi R^2\right]}{3R}\left(T_1 - T_2\right)$

$= \pi R k_0\left(T_1 - T_2\right)$

(Q) $\dfrac{\Delta Q}{\Delta t} = \dfrac{4\pi k R_1 R_2}{\left(R_2 - R_1\right)}\left(T_1 - T_2\right)$

$= \dfrac{4\pi k_0 3R.R}{\left(3R - R\right)}\left(T_1 - T_2\right) = 6\pi k_0 R\left(T_1 - T_2\right)$

(R) $\dfrac{\Delta Q}{\Delta t} = \dfrac{2\pi k_0 1}{\ln\left(\dfrac{R_2}{R_1}\right)}\left(T_1 - T_2\right) = \dfrac{4\pi k_0 R}{\ln 2}\left(T_1 - T_2\right)$

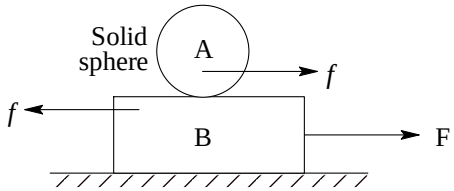
(S) $\dfrac{\Delta Q}{\Delta t} = -\pi R^2 k.\dfrac{dT}{dx} = -\pi R^2 k_0\left(1 + \dfrac{x}{3R}\right)\dfrac{dT}{dx}$

$\therefore \dfrac{\Delta Q}{\Delta t} = \displaystyle\int_0^{3R}\frac{dx}{\left(1 + \dfrac{x}{3R}\right)} = -\pi R^2 k_0\int_{T_1}^{T_2} dt$

$$\therefore \frac{\Delta Q}{\Delta t} = \frac{\pi k_0 R\left(T_1 - T_2\right)}{3\ln 2}$$

9. C

Sol. (P)



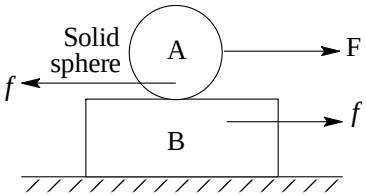
$f = ma_A$

$F - f = ma_B$

$a_A + \propto R = a_B$

$fR = \frac{2}{5}MR^2 \propto$

(Q)



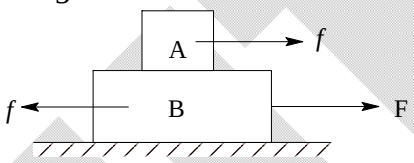
$F - f = ma_A$

$F = ma_B$

$a_A - \propto R = a_B$

$fR = \frac{2}{5}MR^2 \propto$

(R)



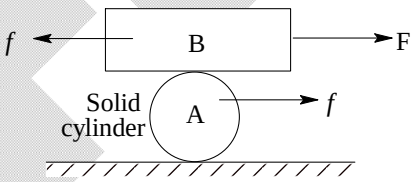
$f = ma_A$

$F - f = ma_B$

$a_A = a_B$

$F - f = ma_B$

(S)



$f = ma_A$

$a_B = a_B + \propto R$

$fR = \frac{MR^2}{2} \propto$

10. D

Sol. For step up transformer maximum value of $\frac{V_{secondary}}{V_{primary}}$ is $\frac{1000}{200}$

For step down transformer maximum value of $\frac{V_{secondary}}{V_{primary}}$ is $\frac{800}{1000}$

For step up transformer minimum value of $\frac{V_{secondary}}{V_{primary}}$ is $\frac{1000}{800}$

For step down transformer minimum value of $\frac{V_{secondary}}{V_{primary}}$ is $\frac{200}{1000}$

11. A

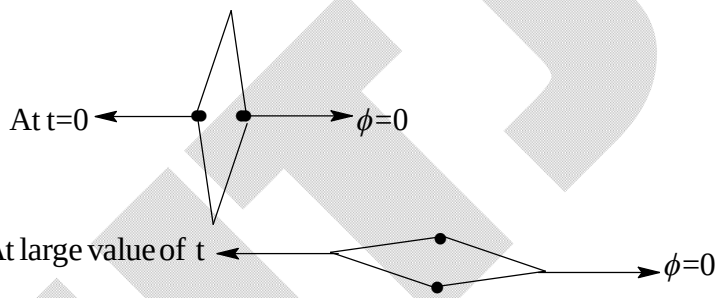
Sol. (1) $V = \frac{kQ}{2R^3}\left(3R^2 - x^2\right)$ *and* $V = \frac{kQ}{x}$

(2) $q = q_0\left(1 - e^{-t/\tau}\right) \Rightarrow \frac{dq}{dt} = i = \frac{q_0 e^{-t/\tau}}{\tau}$

$\Rightarrow q = q_0\left(1 - \frac{i\tau}{q_0}\right) \Rightarrow q = q_0 - i\tau$

(3) $x_0 x_i = f^2$

(4)



At large value of t

(5) $K.E = \frac{1}{2} I\omega^2$

$K.E \propto \omega^2$

SECTION – B

12. 4

Sol. The given wave is superposition of 3 wave with frequency, $\omega_0, \omega + \omega_0$ & $\omega_0 - \omega$

$\omega_{max} = (\omega_0 + \omega)$

$\therefore E_{max} = h\nu_{max} = h\frac{(\omega_0 + \omega)}{2\pi}$

$h\nu_{max} = KE_{max} + \phi \Rightarrow \phi = 4eV$

13. 5

Sol. $\frac{dN}{dt} = \lambda N_0$

$$t = \frac{4,5}{300}\min$$

$$= \frac{1}{200}\min$$

$$= 5\times10^{-2}\ \min$$

14. 4

Sol. $$F - \frac{B\ell}{r}\left(B\ell v - E\right) = m\frac{dv}{dt}$$

$$\Rightarrow F = -mv_0\omega\sin\omega t + \frac{B^2\ell^2}{r}v_0\cos\omega t$$

$$-\frac{B\ell E}{r} \Rightarrow P = Fv$$

$$< P >= \frac{B^2\ell^2 v_0^{\ 2}}{2r}$$

15. 5

Sol. The open end of pipe has pressure node and displacement antinode, so, let open end be x = 0, then particle displacement amplitude will be.

$$s = s_0\cos kx = \frac{\Delta p_m}{Bk}\cos kx \ \&\ k = \frac{2\pi}{\lambda}$$

For sound wave in a gas, $B = \gamma P$ (adiabatic process)

So, $$s = \frac{1.4Pa}{\left(\gamma\times10^5\,Pa\right)\times\frac{2\pi}{\lambda}}\cos\left(\frac{2\pi}{\lambda}x\right)$$

Diatomic gas, $\gamma = \frac{7}{5}$, 1st harmonic, $\frac{\lambda}{2} = L = \pi m\ or\ \lambda = 2\pi m$

$$x = \frac{\pi}{3}m$$

We get, $s = 5\times10^{-6}m$

16. 6

Sol. $$r = \frac{2.4\sqrt{3}}{2}$$

$$T_2\cos30° + T_1\cos30° + mg\cos30° = \frac{mv^2}{r}$$

$$T_2\sin30° + mg\sin30° = T_1\sin30°$$

For just slack condition $T_2 = 0$

$$T_1 = mg$$

$$v = 6\,m/s$$

17. 8

Sol. $$l = \frac{\pi u}{2}\sqrt{\frac{m}{2k}}$$

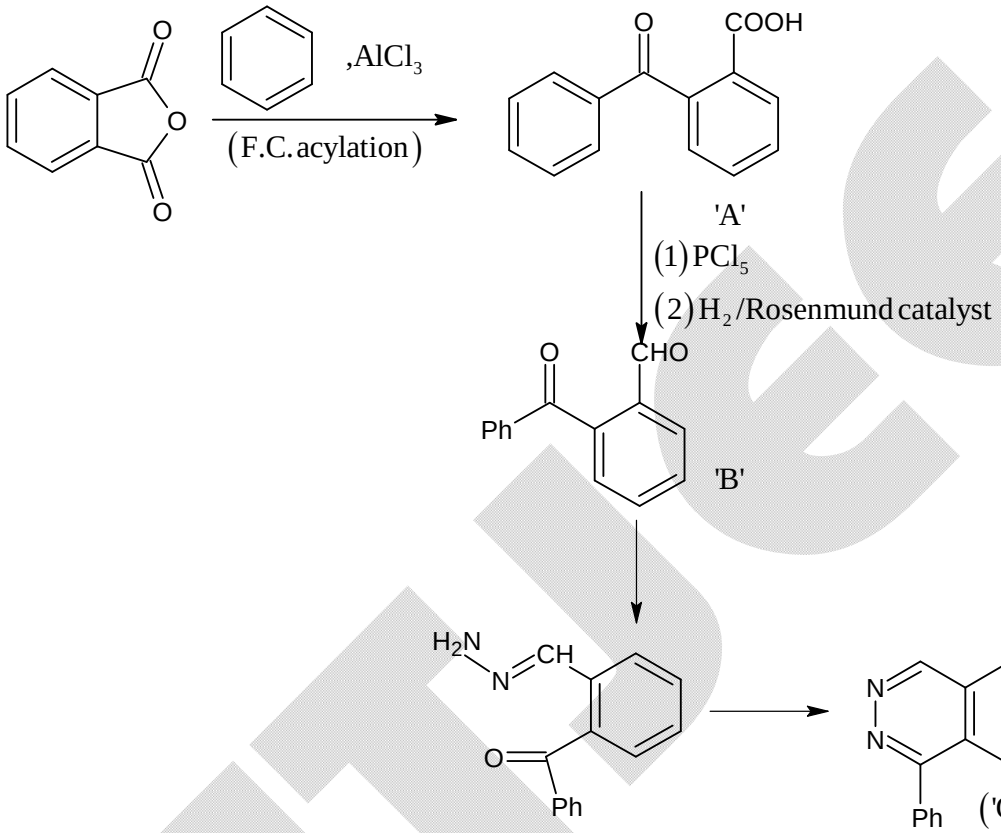
Chemistry

PART – II

SECTION – A

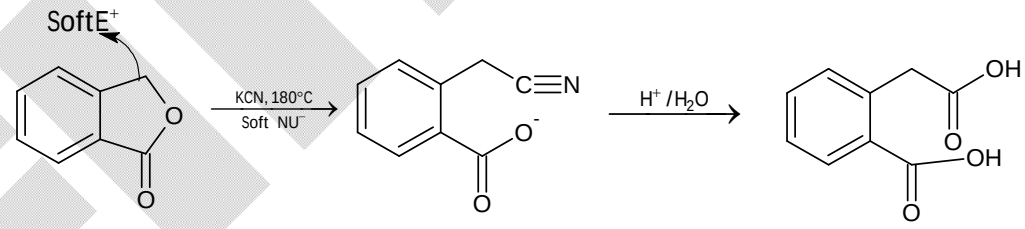
18. AB

Sol.

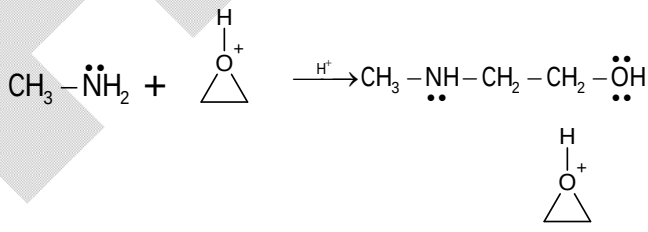


19. ABD

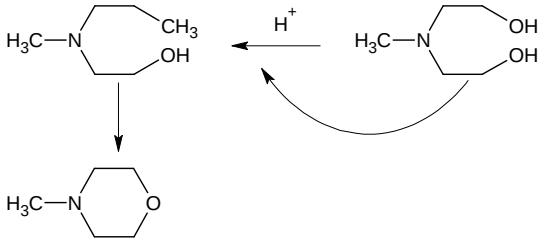
Sol. (A)



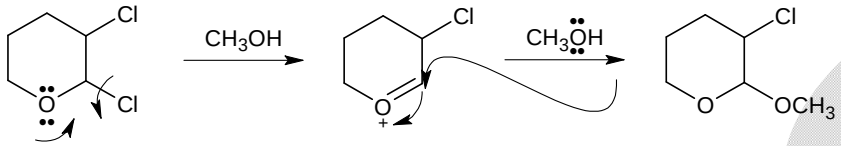
(B)



(C)

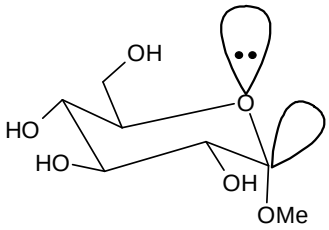


(D)



20. AC

Sol. The two -OH groups forms acetal with Ph – CHO and results into six-member ring in chair conformation.



interactions make axial anomeric position more stable. i.e. anomeric effect.

$\sigma^{*}C-OM.O.$

21. D

Sol. A higher halogen cannot oxidises lower halides because oxidising power $F_2 > Cl_2 > Br_2 > I_2 > At_2$. Solubility in liquid NH_3 decreases AgCl > AgBr > AgI > AgAt.

22. C

Sol. Initially $P_m = P^o_X.X_X + P^o_Y.X_Y$

$$550 = P^o_X\left(\frac{1}{1+3}\right) + P^o_Y\left(\frac{3}{1+3}\right)$$

or $P^o_X + 3P^o_Y = 2200$ ...(i)

When 1 mole of Y is further added to it

$P_m = P^o_X.X_X + P^o_Y.X_Y$

$$560 = P^o_X\left(\frac{1}{1+4}\right) + P^o_Y\left(\frac{4}{1+4}\right)$$

$\therefore P^o_X + 4P^o_Y = 2800$ ...(ii)

By eqs. (i) and (ii)

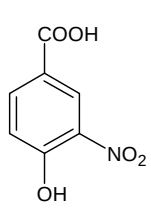
$P^o_X = 400$ mm and $P^o_Y = 600$ mm.

23. B

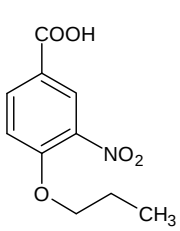
Sol. HOMO is the highest occupied molecular orbital and paramagnetic species have unpaired electrons. Use molecular orbital energy diagrams.

24. B

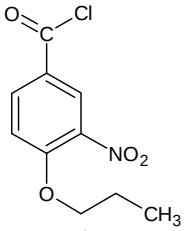
Sol.



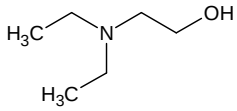
'A'



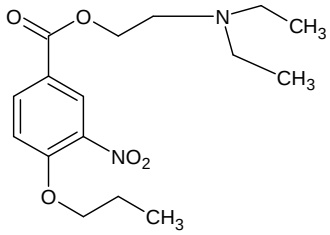
'B'



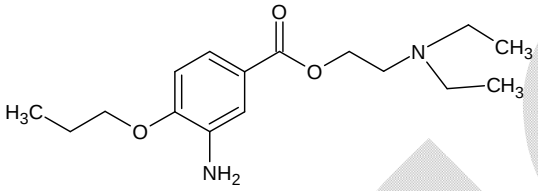
'C'



'D'



'E'



'F'

25. C

26. B

27. B

28. D

Sol. $2S_2O_4^{2-} + H_2O \longrightarrow S_2O_3^{2-} + 2HSO_3^-$

$S_2O_3^{2-} + 2H^+ \longrightarrow SO_2\uparrow + S\downarrow + 2H_2O$

$2S_2O_8^{2-} + 2H_2O \longrightarrow 4SO_4^{2-} + 4H^+ + O_2\uparrow$

$SCN^- + H_2SO_4 + H_2O \longrightarrow COS + NH_4^+ + SO_4^{2-}$

SECTION – B

29. 6

Sol. sp^3d^2, d^2sp^3, dsp^2, sp^3d^3 involve $d_{x^2-y^2}$ orbital in Hybridisation.

$\left[Cr(NH_3)_6\right]^{3+}$, XeF_5^-, $\left[Ni(CN)_4\right]^{2-}$ IF_5, $\left[Pt(Br)_2(en.)\right]$, $\left[Ni(dmg)_2\right]$

30. 5

Sol. Theoretical

31. 25

Sol. Avg. oxidation state of iron = 16/7

Let fraction of Fe^{2+} ions is x. 2x+3(1-x) = 16/7

Fe^{2+} : Fe^{3+} = 5:2 x = 5/7

For every missing of one Fe^{2+} ion, there would be 2 Fe^{3+} ions.

% of cationic vacancies $\frac{1}{8}\times 100 = 12.5\% = a$

$'2a' = 25$

32. 482

Sol. 10 mol of SO_2 in 10^6 mol of air or 10^{-5} mol of SO_2 in 1 mol of air.

$\left[SO_2\right]$ dissolved in water $\alpha\ p_{SO_2}$

$\left[SO_2\right]$ in rainwater = $1.5\times10^{-5}\ mol/ltr.$

$H_2SO_3 \rightleftharpoons H^+ + HSO_3^-$

$$K_a = 10^{-2} = 0.01 = \frac{C\alpha.C\alpha}{C(1-\alpha)} = \frac{1.5\times10^{-5}\times\alpha^2}{1-\alpha}$$

$1.5\times10^{-3}\alpha^2 + \alpha - 1 = 0 \Rightarrow \alpha \approx 1$

$\left[H^+\right] = 1.5\times10^{-5}$

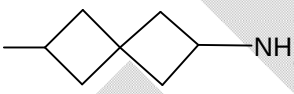
$p^H = 5 - \log 1.5 = 4.82$

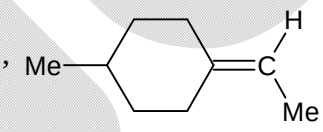
33. 6

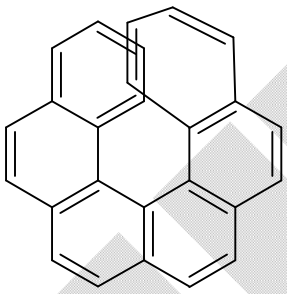
Sol. Degenerate orbitals have equal energy. Number of spherical nodes = n – l – 1.
The wave function is maximum at r = 0 for s-orbital so probability density is maximum at r = 0.
For hydrogen atom energy only depends on principle quantum number (n).

34. 6

Sol. The optical active compounds can be resolved are

trans-cyclooctene , 

, 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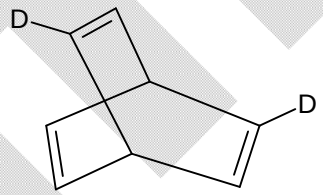

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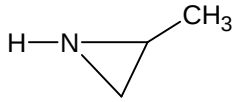


Mathematics

PART – III

SECTION – A

35. B, C

Sol. Let $Y = 2X - 3I$

$\Rightarrow Y^4 = -216X \Rightarrow |Y|^4 = (216)^4.16$

$\Rightarrow |Y| = \pm 432$

$\because 2I - 3X^{-1} = X^{-1}.Y$

So $|\text{adj } X^{-1} Y| = |X^{-1} Y|^3 = \left(\frac{|Y|}{|X|}\right)^3 = \pm 3^9$

Now, we adj (adj A) = $\text{adj}\left(|A|A^{-1}\right) = |A|^2 .A$

So adj (adj X) = 2^8. X

and adj (adj (adj X)) = 2^{28}. X^{-1}

36. A, D

Sol. Eqn. of median through A

$\frac{x-2}{-1} = \frac{y}{1} = \frac{z}{1}$

Let $E(-\lambda+2, \lambda, \lambda)$

$\because \frac{1}{3}$ base area x DE = volume

$\Rightarrow$ DE = 2 units

So AE = $\sqrt{AD^2 - DE^2} = 2\sqrt{3}$

$\Rightarrow \sqrt{(1-\lambda)^2 + (1-\lambda)^2 + (1-\lambda)^2} = 2\sqrt{3}$

$\Rightarrow |\lambda - 1| = 2$

$\Rightarrow \lambda = 3 \text{ or } -1$

So, point E is $(-1, 3, 3)$ or $(3, -1, -1)$

37. B, C

Sol. Solving the quadratic in $\left(\frac{dy}{dx}\right)$

$$\frac{dy}{dx} = \frac{\left(x^2 - y^2\right) \pm \sqrt{\left(x^2 - y^2\right)^2 + 4x^2y^2}}{2xy}$$

$$= \frac{x^2 - y^2 \pm \left(x^2 + y^2\right)}{2xy}$$

$\frac{dy}{dx} = \frac{x}{y}$ or $\frac{dy}{dx} = \frac{-y}{x}$

$x^2 - y^2 = C_1$ or $xy = C_2$

38. B

Sol. $P \sin\theta + Q \cos\theta = \cos\theta + \frac{1}{2}(\cos\theta \sin\theta - \sin\theta \cos\theta) - \frac{1}{4}(\sin 2\theta \sin\theta + \cos 2\theta \cos\theta)$

$= \cos\theta - \frac{1}{4}\cos\theta - \frac{1}{8}\sin 2\theta + \frac{1}{16}\cos 3\theta + ...$

$= \cos\theta - \frac{1}{2}P$

Similarly $P\cos\theta + Q\sin\theta = -2(Q-1)$

Now form above two questions

$P(1 + 2\sin\theta) + 2Q\cos\theta = 2\cos\theta$

and $P\cos^2\theta + Q\cos\theta(2 + \sin\theta) = 2\cos\theta$

subtract the two and rearrange to get

$\frac{P}{Q} = \frac{\cos\theta}{2+\sin\theta} = \frac{2\sqrt{2}}{7}$

$\Rightarrow 7\cos\theta - 2\sqrt{2}\sin\theta = 4\sqrt{2}$

Has two solutions in one period (i.e. $(0, 2\pi)$)

39 C

Sol. $\because \quad \frac{\log b}{\log a} + 6\frac{\log a}{\log b} = \frac{(\log b)^2 + 6(\log a)^2}{\log a.\log b} = 5$

$\Rightarrow (\log b - 3\log a)(\log b - 2\log a) = 0$

$\therefore$ log b = 3 log a or log b = 2 log a

So either $b = a^3$ or $b = a^2$

For the case $b = a^2$, $44^2 = 1936$ and $45^2 = 2025$

$\therefore$ all values of a from 2 to 44 will acceptable.

For the cube $b = a^3$, $12^3 = 1728$ and $13^3 = 2197$

Then all values of a from 2 to 12 acceptable.

There are $44 - 2 + 1 = 43$ possibilities for

Square cube and $12 - 2 + 1 = 11$ possibilities for the cube case

$\therefore$ Total ordered pairs = 43 + 11 = 54

40. C

Sol. ${}^nC_{2k}$ = Coeff. Of x^k in $\left(1+\sqrt{x}\right)^n$

= Coeff. Of x^k in $\left(2+\sqrt{x}-1\right)^n$

= Coeff. Of x^k in $\sum_{r=0}^{n} {}^nC_r \left(\sqrt{x}-1\right)^r .2^{n-r}$

$= \sum_{r=0}^{n} (-1)^{r-2k} . {}^nC_r . {}^rC_{2k} . 2^{n-r}$

Similarly, for $\left(3+\sqrt{x}-2\right)^n$

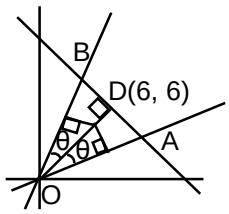
41. A

Sol. Incentre will lie on OD

$OD \equiv y = x$

Let inradius is 'r' then incentre $I\left(6-\frac{r}{\sqrt{2}}, 6-\frac{r}{\sqrt{2}}\right)$

$\because \quad \tan 2\theta = 2\sqrt{2}$



$\Rightarrow \sin\theta = \frac{1}{\sqrt{3}} = \frac{r}{6\sqrt{2}-r}$

$r = \frac{6\sqrt{2}}{\sqrt{3}+1} = 3\sqrt{2}(\sqrt{3}-1) \Rightarrow I(9-3\sqrt{3}, 9-3\sqrt{3})$

Area of incircle = $\pi r^2 = 36(2-\sqrt{3})\pi$

$\text{Cos}2\theta + \cos(90 - \theta) + \cos(90 - \theta) = 1 + \frac{r}{R}$

$\Rightarrow \frac{r}{R} = \frac{1}{3} + \frac{2}{\sqrt{3}} - 1$

$= \frac{2}{3}(\sqrt{3}-1) \Rightarrow \frac{R}{r} = \frac{3}{4}(\sqrt{3}+1)$

$\because$ AB = 2OD tan θ = $2.6\sqrt{2}.\frac{1}{\sqrt{2}} = 12$

$\Delta = \frac{1}{2}AB.OD = 36\sqrt{2}$

$s = \frac{\Delta}{r} = \frac{36\sqrt{2}}{3\sqrt{2}(\sqrt{3}-1)} = 6(\sqrt{3}+1)$

42. B

Sol. (P) $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

And $3z_0 = z_1 + z_2 + z_3$

So $\frac{z_1^2 + z_2^2 + z_3^2}{z_0^2} = 3$

(Q) Clearly $z = -1 - i$

So = $z + \bar{z} + z\bar{z} = 0$

(R) $|iz + z_0| = |z - iz_0| \le |z - 1| + |i - iz_0| = 2 + 5 = 7$

(S) Let $w = 1 - 4z \Rightarrow z = \frac{1-w}{4}$

And $|z - z_0| = \frac{1}{2}$

So $\left|\frac{1-w}{4} - z_0\right| = \frac{1}{2}$

$|w - (1 - 4z_0)| = 2$

So locus w will be a circle of radius 2

43. D

Sol. (P) Let $\int_{-\pi}^{\pi} \frac{x^2}{1+\sin x + \sqrt{1+\sin^2 x}} dx$ (replaced x by $-x$)

$I = \int_{-\pi}^{\pi} \frac{x^2}{1-\sin x + \sqrt{1+\sin^2 x}} dx; 2I = \int_{-\pi}^{\pi} x^2 \left[\frac{1}{1+\sin x + \sqrt{1+\sin^2 x}} + \frac{1}{1-\sin x + \sqrt{1+\sin^2 x}}\right] dx$

$= \int_{-\pi}^{\pi} x^2 dx; 2I = \int_{-\pi}^{\pi} x^2 dx; I = \frac{\pi^3}{3}$

(Q) Let y = $y = \frac{1}{x}$, dy = $-\frac{1}{x^2}dx$ then $\frac{dx}{x^2-x+1} = \frac{dy}{y^2-y+1}$; $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2}$; $\tan^{-1}\frac{1}{x} = \cot^{-1}x$

$\therefore \quad \frac{1}{2}\left(\int_{1/2}^{2} \frac{\tan^{-1}}{x^2-x+1}dx + \frac{\frac{\pi}{2}-\tan^{-1}x}{y^2-y+1}dx\right) = \frac{\pi}{4}\int_{1/2}^{2}\frac{dx}{x^2-x+1} = \frac{\pi^2\sqrt{3}}{18}$

(R) $\int_0^{\frac{\pi}{2}} \frac{\frac{\cos x}{x}}{\left(\frac{\sin x}{x}\right)^2}dx$; Put $\frac{\sin x}{x} = t$

$\frac{x\cos x - \sin x}{x^2}dx - dt; \int_1^{\frac{2}{\pi}} \frac{dt}{1+t^2}; \left[\tan^{-1}x\right]_1^{2/\pi}; \tan^{-1}\frac{2}{\pi} - \frac{\pi}{4}$

(S) As function is odd

44. A

Sol. (P) Let $f(x) = (x+1)^5 - 2(x^5+1)$

$\Rightarrow f'(x) = 5(x+1)^4 - 2(5x)^4 = 5[(x+1)^4 - 2x^4]$

$f'(x) = 0$

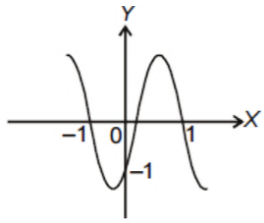
$\Rightarrow \left(\frac{x+1}{x}\right)^4 = 2$

$\Rightarrow 1 + \frac{1}{x} = \pm(2)^{1/4}$

$\Rightarrow \frac{1}{x} = -1 \pm \sqrt[4]{2}$

$\Rightarrow x = \frac{1}{\left(-1\pm\sqrt[4]{2}\right)}$ i.e., $\frac{1}{-1-(2)^{1/4}}$ and $\frac{1}{-1+(2)^{1/4}} = \alpha, \beta$

(Say)



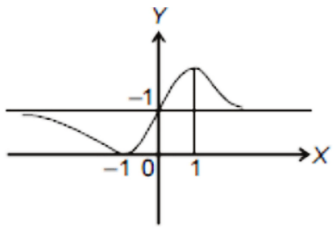
And $f(-\infty) = \lim_{x\to\infty}(x+1)^5 - 2(x^5+1)$

$= \lim_{x\to\infty} x^5\left[\left(1+\frac{1}{x}\right)^5 - 2\left(1+\frac{1}{x^5}\right)\right] = \lim_{x\to\infty} x^5[1-2] = \infty$

$\because$ f'(x) = 0 has two real roots and f(x) is polynomial of 5th degree and f'(α) and f'(β) = 0 but $f'(\alpha) \neq 0$ $f'(\beta) \neq 0$

$\Rightarrow$ f(x) = 0 has exactly 3 distinct real roots

(Q) $f(x) = \frac{(x+1)^4}{x^4-x^3+x^2-x+1} = \frac{(x+1)^4}{\left(\frac{1-(-x)^5}{1-(-x)}\right)} = \frac{(x+1)^5}{(1+x^5)}$



$f'(x) = \dfrac{\left(x^5+1\right)5(x+1)^4-(x+1)^5\left(5x^4\right)}{\left(x^5+1\right)^2}$

$= \dfrac{5(x+1)^4(1-x)(1+x)\left(1+x^2\right)}{\left(1+x^5\right)^2}$

$f'(x) = 0$ at $x = -1, 1$

Also $f(x) = 0$ at $x = -1$ and $f(x) \geq 0$

$f'(x) < 0$ for $x \in (-\infty,-1)\cup(1,\infty)$ and $f'(x) > 0$ for $x \in (-1, 1)$

f_{max} (absolute) = max $\{f(-\infty), f(1), f(\infty)\}$

But $f(-\infty) = f(\infty) = 1$ and $f(1) = 16$

f_{max} (absolute) = 16

(R) $f(x) = ab\sin x + \sqrt{1-a^2}\cos x + c; |a| < 1$

$f(x) \in \left[c-\sqrt{a^2b^2+\left(1-a^2\right)}, c+\sqrt{a^2b^2+1\left(1-a^2\right)}\right]$

$\Rightarrow \left(f_{max}-f_{max}\right) = 2\sqrt{a^2b^2+\left(1-a^2\right)}, c+2\sqrt{2\left(b^2-1\right)+1}$

Now $a^2\left(b^2-1\right)+1 \leq \dfrac{[(0)^2-4(b^2-1)(1)]}{4(b^2-1)}$

$\left(\because ax^2+bx+c \leq \dfrac{-D}{4a} \text{ for } a<0\right)$

$\Rightarrow a^2\left(b^2-1\right)+1 \leq 1 \text{ at } a<0$

$\Rightarrow \left(f_{max}-f_{min}\right) \leq 2$

(S) $u^2 = (4+1) + 2\sqrt{4\cos^4\theta+4\sin^4\theta+17\sin^2\theta\cos^2\theta}$

$\Rightarrow u^2 = 5 + 2\sqrt{4\left(\sin^2\theta+4\cos^2\theta\right)^2+9\sin^2\theta\cos^2\theta}$

$\Rightarrow u^2 = 5 + 2\sqrt{4+\dfrac{9}{4}\sin^2 2\theta}$

$\Rightarrow u^2_{min} = 5+2(2) = 9 \text{ and } u^2_{max}$

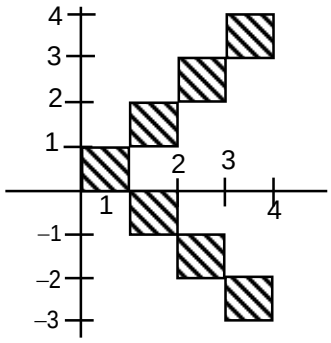
$5 + 2\sqrt{4+\dfrac{9}{4}} = 5+2\left(\dfrac{5}{2}\right) = 10$

$\Rightarrow \left(u^2_{max}-u^2_{max}\right) = (10-9) = 1$

45. B

Sol.

(P)

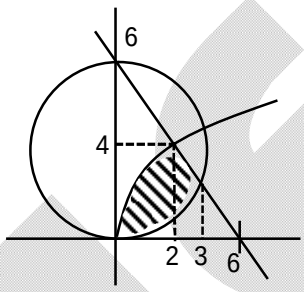


Required area = 7 units

(Q) Required area

$$= \int_0^2 \sqrt{8x}\, dx + \int_2^3 (6-x)\, dx - \int_0^3 \left(3-\sqrt{9-x^2}\right) dx$$

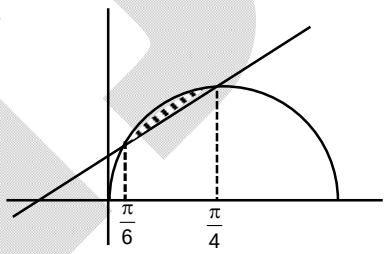
$$= \frac{27\pi - 2}{12}$$



(R) Required area

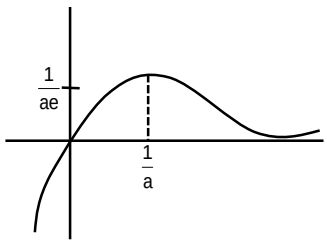
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin x\, dx - \frac{1}{2}\left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)\frac{\pi}{12}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{2} - \frac{\left(\sqrt{2}+1\right)\pi}{48}$$



(S) $\int_0^\infty x.e^{-ax}\, dx = \frac{1}{9}$

$$= \frac{1}{a^2} = \frac{1}{9} \Rightarrow a = 3$$



SECTION – B

46. 4

Sol. Clearly (p, q) and (q, p) are reflection of each other in the line y = x.

Hence chord must be of the form

$y = -x + \lambda$

Solving with the parabola

$-x + \lambda = ax^2 + bx$

$\Rightarrow\ ax^2 + (b+1)x - \lambda = 0$ …(i)

Again solving parabola and chord

$y = a(\lambda - y)^2 + b(\lambda - y)$

$\Rightarrow ay^2 - (2\lambda a + b + 1)y + a\lambda^2 + b\lambda = 0$ ...(ii)

Now (i) and (ii) should have same roots p and q

$\Rightarrow b + 1 = -(2\lambda a + b + 1)$ and $-\lambda = a\lambda^2 + b\lambda$

$\Rightarrow \lambda = -\left(\frac{b+1}{a}\right)$

For roots of (i) to be real and distinct

$(b+1)^2 + 4a\lambda > 0$

$\Rightarrow (b+1)^2 - 4a\left(\frac{b+1}{a}\right) > 0$

$\Rightarrow (b+1)(b-3) > 0$

$\Rightarrow b \in (-\infty, -1) \cup (3, \infty)$

47. 990

Sol. Let x = k + r, where k is non-negative integer and $r \in [0, 1)$

Given equation gives

$3kr(k + r) = [3kr(k + r) + r^3]$

So, there is a solution to the equation for every x for which 3kr (k + r) is an integer, where $K \in [0, 9]$ and

$r \in [0, 1)$

Now for a fixed k, as r increase from 0 to 1, 3kr (k + r) increase from 0 to 3k (k + 1), so it assumes exactly

3k (k + 1) integral values

Hence total number of solutions = $\sum_{k=1}^{9} 3k(k+1)$

48. 2

Sol. $\left(\sqrt{10+\sqrt{n}} - \sqrt{10-\sqrt{n}}\right)^2 = 2\left(10 - \sqrt{100-n}\right)$

$\Rightarrow \sqrt{10+\sqrt{n}} - \sqrt{10-\sqrt{n}} = \sqrt{2}\sqrt{10-\sqrt{100-n}}$

$\Rightarrow \sum_{n=1}^{99}\sqrt{10+\sqrt{n}} - \sum_{n=1}^{99}\sqrt{10-\sqrt{n}} = \sqrt{2}\sum_{n=1}^{99}\sqrt{10-\sqrt{100-n}}$

$\Rightarrow \frac{\sum_{n=1}^{99}\sqrt{10-\sqrt{n}}}{\sum_{n=1}^{99}\sqrt{10+\sqrt{n}}} = \sqrt{2}-1$

49. 2019

Sol. $3^\gamma \alpha + 2.3^{\gamma-1}\alpha = 3^{\gamma-1}.(5\alpha) = 3^{\gamma-1}.90^\circ$

$\Rightarrow 3^\gamma \alpha = 3^{\gamma-1}.90 - 2.3^{\gamma-1}\alpha\ 3$

$\Rightarrow \sin(3^\gamma \alpha) = (-1)^{\gamma-1}\cos(2.3^{\gamma-1}\alpha)$

$\Rightarrow a_n = \sum_{\gamma=1}^{n}(-1)^{\gamma-1} = \begin{cases} 1 \text{ if } n \in \text{odd} \\ 0 \text{ if } n \in \text{even} \end{cases}$

Now,

$2^{\gamma}\beta + 3.2^{\gamma-1}\beta = 2^{\gamma-1}.180^{o}$

$\Rightarrow \cos\left(2^{\gamma}\beta\right) = \begin{cases} -\cos\left(3.2^{\gamma-1}\beta\right) & \text{for } \gamma = 1 \\ \cos\left(3.2^{\gamma-1}\beta\right) & \text{for } \gamma \neq 1 \end{cases}$

$\Rightarrow b_n = n - 2$

50. 14

Sol. Let $a = \cot\theta$

So, clearly $b = \cot 2\theta, c = \cot 4\theta, d = \cot 8\theta \text{ and } a = \cot 16\theta = \cot\theta$

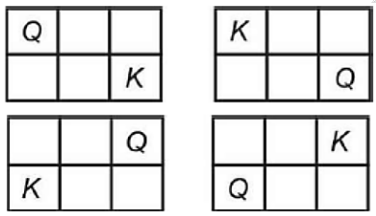
$\Rightarrow 16\theta = n\pi + \theta$

$\Rightarrow \theta = \frac{n\pi}{15}$ where n = 1, 2, 3, 4,...14

51. 20

Sol. First we count the case when knight can attack on queen. In each rectangle of dimension 2 x 3, there are 4 possible arrangements of queen and knight as down

So, number of such cases = (2 x 6 x 7) x 4 = 336

Now, we count the cases when queen attack on knight. For every position of queen there are 14 positions of knight (in same row or column). Also queen can attack diagonally in $2\left[{}^{8}C_{2} + 2\left({}^{7}C_{2} + {}^{6}C_{2} + {}^{2}C_{2}\right)\right]\underline{|2}$ ways

Hence $P = 1 - \frac{336 + 64 \text{ x } 14 + 560}{{}^{64}C_{2}.\underline{|2}} = \frac{5}{9}$